He then applied this generic solution to the problem by solving

$$x^2 + xy + y^2 = (7^3)^2 = 343^2.$$

There are two positive integer solutions to this equation: $(x, y) = (18, 1)$ and $(x, y) = (14, 7)$. With these solutions it was possible for him, by retracing his steps, to obtain two sets, wherein each set contains two isosceles triangles with sides (a, a, b) and (c, c, d), and for which the triangles have the same perimeter, the same area, and for which $\dfrac{b^2 + bd + d^2}{b + d} = 7^6$.

Also solved by Bruno Salgueiro Fanego, Viveiro, Spain; Ed Gray, Highland Beach, FL; David Stone and John Hawkins (jointly), Georgia Southern University, Statesboro GA, and the proposer.

- **5249:** *Proposed by Tom Moore, Bridgewater State University, Bridgewater, MA*

(a) Let n be an odd positive integer. Prove that $a^n + b^n$ is the square of an integer for infinitely many integers a and b.

(b) Prove that $a^2 + b^3$ is the square of an integer for infinitely many integers a and b.

Solution 1 by Arkady Alt, San Jose, CA

(a) Let $a = x(x^n + y^n), b = y(x^n + y^n)$ where $x, y \in \mathcal{N}$ then

$$a^n + b^n = x^n(x^n + y^n)^n + y^n(x^n + y^n)^n = (x^n + y^n)^{n+1}$$

and, since $n = 2m - 1, m \in \mathcal{N}$ then

$$a^n + b^n = ((x^n + y^n)^m)^2.$$

(b) We will show that equation $a^2 + b^3 = c^2$ have infinitely many solutions in integers. Assuming that $c = 2a$ we obtain $b^3 = 3a^2$. Let $a = 3t^3, t \in \mathcal{Z}$ then

$$b^3 = 3 \cdot 9t^6 \iff b = 3t^2.$$

Thus, for $(a, b) = (3t^3, 3t^2)$, where t is any integer we have

$$a^2 + b^3 = 9t^6 + 27t^6 = 36t^6 = \left(6t^3\right)^2.$$

Solution 2 by Pat Costello, Eastern Kentucky University, Richmond, KY

(a) Let n be an odd positive integer. Let $a = 2 \cdot 2^{2j}$ and $b = 2 \cdot 2^{2j}$ for an arbitrary positive integer j. Then

$$\begin{aligned} a^n + b^n &= (2 \cdot 2^{2j})^n + (2 \cdot 2^{2j})^n \\ &= 2^n \cdot 2^{2nj} + 2^n \cdot 2^{2nj} \\ &= 2 \cdot \left(2^n \cdot 2^{2nj}\right) \\ &= 2^{n+1} \cdot 2^{2nj} \end{aligned}$$